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Students' Concept Image and Its Impact on Reasoning towards the Concept of the Derivative

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Abstract: The aim of this study was to identify and classify the student's concept image and its influence on the reasoning of the problem-solving of the derivative. The research used a qualitative description approach and used eight research subjects. From the answers collected upon the given problems, we obtained several variations of students' concept images, thus it showed how students' concept image influenced the reasoning. In order to clarify and classify the characteristic of the obtained answers, we summarized there were three categories of the concept image of the derivative, namely symbolically related to a basic formula of the derivative of a function, limit of the ratio of difference value of the functions, and the properties of the derivative of the functions. Furthermore, our study suggested that each student's concept image affecting the reasoning of the derivative. In addition, we found some misperceptions in answering the problem and misconception in the use of the basic formula of the derivative of the functions among the students' answers.

Keywords: *Concept image, perception, mathematical reasoning, conception, misconception.*

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Introduction

Mathematics is a daily ability that could develop logic skill and it is thought to be important to enhance the reasoning skill in mathematics, including in analyzing and abstracting as well as to use an appropriate strategy to solve the problem. Even in Pahrudin et al. (2019), also stated that the students with higher learning motivation is considered to have higher mathematical reasoning ability than students with lower learning motivation. According to Lithner (2008), the reasoning was defined as " A line of thought adopted to produce assertions and reach conclusions in task solving". Lithner distinguished mathematical reasoning into *imitative reasoning* and *mathematical creative founded reasoning*. Imitative reasoning is often used as reasoning in proving theorems, while mathematical creative founded reasoning is commonly used as reasoning on problem-solving.

Understanding the derivative is a need by mathematics students or students who are studying calculus at the undergraduate level. In general, encouraging students to learn calculus in mathematics will likely support their ability to solve problems since it strengthens the understanding and reasoning.

Even the students need to be encouraged in mathematical proving activities due to its advantage in constructing students' reasoning. The proof is derived from deductive reasoning, not from inductive or empirical arguments. In this regard, reasoning refers to using logical and coherent arguments to draw conclusions, inferences, or judgments (Ross,1997) in Aineamani (2011). Deductive reasoning is a logical thinking process from true known premises then derived the conclusions (Yopp, 2009). A diagram method to describe deductive reasoning consists of three components, *both teacher and students provide case as data, result* as a claim that should be proved by students*, the rule* is provided by teacher (Toulmin, 2003). The Toulmin's diagram is described in Figure 1.

Reasoning can also be understood as a process of drawing a conclusion based on the evidence and the assumptions (Ross, 1997) in Aineamani (2011). Based on Mckenzie (2001) and Kilpatrick et al. (2001), understanding in mathematics is based on mathematical reasoning, and it is beneficial for solving problems. Even, Mueller and Maher (2009) stated that the reasoning is very important for developing the students' mathematical knowledge. Therefore,

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students can use mathematical reasoning and apply mathematical ideas to develop their problem-solving skills. Without reasoning allows students the difficulty of producing the correct conclusions. By applying the reasoning, the students will be able to generalize and make a conjecture and justification logically based on already-understood ideas in their problem-solving activities. In mathematical problems-solving, in fact, in real-world problems, people often need reasoning to get a satisfactory solution. In this regard, the reasoning is a thinking process to draw conclusions from true known premises. There are three components of the reasoning process: *conjecturing, generalizing*, *justifying*. *Conjecturing* is the alleged statements that can be true or false; *Generalizing* as induction from something particular to something general by looking at the common things or extending the reasoning beyond the range in which it originated; *Justification is* a logical argument based on already-understood ideas (Lannin et al., 2011).

Meanwhile, the students' reasoning in mathematics often used the concept image in students' minds rather than on a formal mathematical concept (Vinner & Dreyfus, 1989). The students' understanding of a mathematical concept is the construction of two mental entities in the mind of the students: *concept image* and *concept definition* (Vinner & Hershkowitz, 1980; Tall & Vinner, 1981). Based on Vinner (1983), mathematical reasoning can be described as an interplay between concept image and concept definition, it is shown in Figure 2.

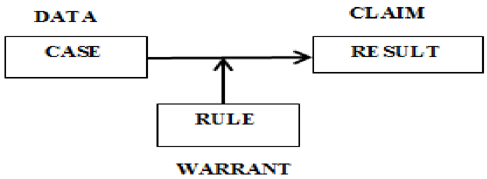


Figure 1. Toulmin's diagram of deductive reasoning

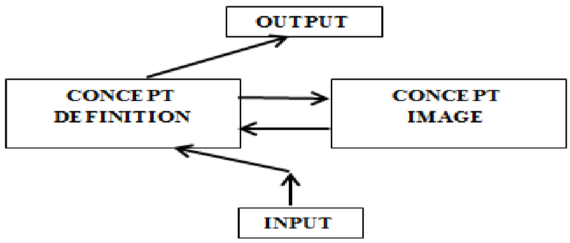


Figure 2. Interplay between concept definition and concept image

According to Tall and Vinner (1981), *Concept image* is defined as "total cognitive structures including all mental pictures, properties and processes associate of the concept" (p. 152). Therefore, from this definition, all mental pictures include words and symbols or pictures associated with the mathematical concept in the individual's mind. *Concept definition* means a statement in words used to specify the concept. Nevertheless, in mathematics education, definitions of concepts are important issues to be addressed. There may be difficulties encountered by some mathematics educators when making definitions (Altun & Konyalioglu, 2019).

Concept image is personal, dynamic and continue to evolve depending on daily student's experiences for learning in the classroom, activities in problem-solving, mathematical projects, or daily mathematics activities. The concept image is not something verbal in students' mind associated with the concept. However, it can describe the cognitive structure of the students' understanding that associated with the concept, that includes all mental pictures, properties, and processes that may be associated with the concept. The knowledge that the students have been obtained in or outer the classroom will be brought in their mind in the form of mental pictures, including the properties, and processes associated with the concept. To analyze students' understanding of the concept, it can be used "constructs" of students' concept image (Vinner, 1991). Even, students' beliefs in solving of the calculus problems are influenced by their concept image of the concepts of calculus (Nurwahyu & Tinungki, 2020). In general, university mathematics students or mathematics education students have concept images in proving mathematics (Moore, 1994).

The concept image can be seen from students in making explanations, procedures, or sequence of steps. The formation of a concept image and concept definition may be a result of students memorizing a formal definition without connecting meaning to it (Edwards & Ward, 2004; Vinner, 1983). Usually, the students like to memorize this concept definition rather than understand it. In this regard, the formal mathematical concept definition is the mathematical concept accepted by the mathematician or mathematical community. In this regard, the formal mathematical concept definition is the mathematical concept accepted by the mathematician or mathematical community. Furthermore, a student's concept image may or may not include the formally correct mathematical definition, because the students' concept image is defined as a personal concept definition, and it can be different from the formal concept definition. Therefore, the student's ability to understand a mathematical concept can be measured from the wide gap between the student's concept image and the formal mathematical concept. If the gap between students' concept image and the formal mathematical concept is closer, students' understanding of the mathematical concept is more correct.

Students in the first semester generally obtain calculus as an exact science introductory course. Calculus includes the limit of the function, continuity, derivative, and integral. The derivative is very important for undergraduate mathematics students or other undergraduate students, because the concept of the derivative is useful to solve certain quantity problems like its application in physics, engineering, or economic.

In Indonesia, the current problem in teaching calculus (derivatives) in undergraduate students is that teachers have difficulty embedding derivative concepts in their students, students prefer to memorize and use formulas of the derivative such that students are only able to solve derivative problems that are directly related to these formulas (using procedural knowledge and solving of the problem with finding similarity by previous problems). Meanwhile, problems of derivative whose answers cannot be directly related to the formula tend not to be successful in solving them because the students lack making connections and lack of imagining concepts in other areas. Related to it, Tall (1993, 1997) stated that difficulties in learning derivatives among undergraduate students are due to their weakness in solving problems involving the concepts. For example, students will face a difficulty if asked to solve $\frac{df(x)}{dg(x)}$ if $f(x) = x|x|$ with $g(x) = x$, or if $f(x) = x^2$ with $g(x) = x^3$. According to Tall (1997), this happens because the students' concept image and concept definition of the derivative is still weak. Thus, the pedagogical aspects of teaching derivative at the undergraduate level are how the teacher encourages students to develop methods in mathematical modeling, enhancing connection skills, and strengthen students' concept images of the concept of the derivative. Therefore, the focus of this study is to investigate the types of students' concept images and their impact on students' reasoning in solving problems related to the concept definition of the derivative.

Generally, the lecturers or teachers teach derivative of function covering the concept of the derivative $y = f(x)$ of a variable *x* is not merely a measure of the rate at which the value of the function y of the function changes with respect to the change of the variable *x*. However, also as a slope or gradient of a tangent line at a point on a curve. They also give a definition of the derivative of a function as a limit of the average rate of change of function $f(x)$ in the form as follows:

$$f'(x) = \frac{df(x)}{dx} = \lim_{h\to 0}\frac{f(x+h) - f(x)}{h}$$

Even sometimes the teachers give in general form $\frac{df(x)}{dg(x)} = \lim_{h\to 0}\frac{f(x+h)-f(x)}{g(x+h)-g(x)}$, for increasing $g(x)$.

In general, when a lecturer teaches derivative of a function, the lecturer explains the notion of the derivative of the functions in the form as follows: $\frac{df(x)}{dx} = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ and its geometric interpretation (Varberg et al., 2007). The lecturer gives some examples for a certain function and some basic formulas of derivative such that the students will familiar use formulas in the practices. However, when observed, some teachers still do not provide interesting examples or challenging exercises to build student's reasoning. Teachers' ability in selecting suitable mathematical examples was strongly related to their mathematics content knowledge for teaching (Rowland et al., 2005). Watson and Mason (2002) stated that learning mathematics can be seen as a process of generalizing from specific examples. Examples are therefore paramount in mathematical teaching and learning. Even according to Kent (2017), responsive teaching involves a deeper exploration into the content of mathematics at the core of the lesson based on the students' understandings and interpretations. For example, how the students solve $\frac{df(x)}{dx^3}$, or $\frac{df(x)}{dg(x)}$ for increasing function $g(x)$. To solve $\frac{dx^2}{dx^3}$, it can be used $\frac{df(x)}{dg(x)} = \lim_{h\to 0}\frac{f(x+h)-f(x)}{g(x+h)-g(x)}$, or $\frac{df(x)}{dg(x)} = \frac{f'(x)}{g'(x)}$, or by using derivative of composition of function $\frac{df(x)}{dg(x)} = \frac{dh(g(x))}{dg(x)}$ (Varberg et al., 2007).

In this study, concept image is a cognitive structure that includes mental pictures, properties, and processes associated with the concept. The aim of this study is to describe and classify the types of the concept image of the derivative and its influence on mathematical reasoning in solving the problem related to the concept of the derivative.

Theoretical Framework

In the research, we drew a theoretical framework for the derivative of Zandieh (2000). Zandieh stated that the concept of the derivative can be seen as a function whose value at any point *x* is the limit of the ratio of differences $f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$. Such that according to (Zandieh, 2000), the derivative has three layers: ratio, limit, and function. We drew the theoretical framework for the concept image based on the notion of concept image (Tall & Vinner, 1981; Vinner, 1983; Vinner & Dreyfus, 1989; Vinner, 1991; Vinner & Hershkowitz, 1980; Sfard, 1991), which draws on the notion of reasoning (Lithner, 2008; Vinner & Dreyfus, 1989). In this case, Vinner and Dreyfus (1989) claimed that the reasoning is the process of thinking in conclusion in the problem-solving activities and the reasoning frequently based on the concept image.

Theoretically, this research method is based on someone who has studied a concept, then, in his/her mind, will keep a concept image (Vinner, 1993). This concept image continues to develop as the person's experience increases, then from the concept image is used to build reasoning in solving problems (Vinner & Dreyfus, 1989).

Student's concept image may or may not include the formally correct mathematical definition. Concept images are not formed merely by definition but by all experiences of students, but concept images are quite personal and experiential. In learning mathematics, students will construct concept images through in some varieties. These include, but are not limited to, classroom instruction, exploration of examples, working problems, and applying the formal definition (Vinner & Dreyfus, 1989). According to Tall and Vinner (1981), the students' reasoning does not base on the concept definition, but on their concept image, even though they realize of these definitions and can recite and explain them.

In this study, although all participants come from some anonymous universities in Sulawesi, Indonesia, they have been familiar with the derivative concept as undergraduate students. They were asked to define that concept in a given task. Upon solving the tasks, there will be various arguments about the concept of the derivative of a function depending on their mathematical understanding of that concept and mathematical reasoning skill. Mathematical understanding is a process that transforms a mathematical object to become a mental object (Sfard, 1991). Meanwhile, objects are stored in students' minds as a concept image associated with the concept (Tall & Vinner, 1981).

The research emphasizes the disclosure of the concept image of a specific concept of a derivative and its impact on reasoning in problem-solving on a specific derivative of a function.

The following research questions are fundamental for this study:

1. What are the concept images created by the students when working with the derivative?
2. Is it possible to classify students' concept images in distinct categories?
3. What is the impact of the concept image on reasoning in problem-solving of the derivative?

Methodology

This part of the study is a qualitative descriptive approach to get a picture of the student concept images of the derivative. The limited number of participants is not statistically sufficient, but it is rather the type of information given in the answers than the quantity being evaluated and analyzed. This study was conducted in Makassar, Indonesia in September – December 2019 for undergraduate students. The undergraduate students who have passed calculus or basic mathematics course I. This study used a descriptive qualitative research approach. Two problems were issued to all participants, and the answers to the problems generate conceptions or misconceptions of the concept of the derivative of functions. The problems were designed to provide students with an opportunity to express thoughts and conceptions of the derivative. The analysis is based on the answers obtained from the given problem.

Participants

The participants of the research are undergraduate students who have taken and passed of Basic Mathematics Course I or Calculus I from four anonymous universities in Makassar, South Sulawesi, Indonesia at the latest in the last two years. The number of students was 60 undergraduate students consist of 39 female students and 21 male students. All participants were voluntary and acquired on 10 – 24 September 2019, through membership of WhatsApp groups of undergraduate students, where they had learned Basic Mathematics Course I or Calculus I at their respective universities in the last two years.

Procedure

From 60 selected participants, the researcher formed a WhatsApp group. In order to reach the goals of the research, two individual questions in the additional instrument of the research were sent to 60 participants of this study via the WhatsApp group on 30 September 2019. The problems were open and not given as mathematical tasks. Each student was given one hour to answer the problems and the written answers were photographed and sent via the WhatsApp groups. The written answers to the problems were then analyzed carefully and thoroughly such that it will generate a number of types the students' answers. From all participants' written answers, the researchers categorized the written answers in some type of answers. It is based on the fact that each student has different experiences, so their concept image may be different. Therefore, researchers will categorize students' answers in order to see the category of the concept image of the students. To obtain deeper information about the reasoning of the students, then from each category of the answer of problem 2 obtained, one participant was selected as the subject for the unstructured written interviewed by the researcher. The unstructured written interview for the subjects was conducted by the researchers via WhatsApp.

Instruments

In descriptive qualitative research, the researchers are the main instrument to obtain a guarantee of the validity and credibility of the data (Lincoln & Guba, 1985). It means that the researchers are the main data collectors and always focus on their research objectives and have many opportunities to make in-depth observations of data on all selected subjects, through written tests, structured or unstructured interviews, and through other documents related to the focus of their research. The researcher created an additional instrument, collected data through the additional instrument, observed and interviewed some needed subjects, and analyzed it. For that, in this study was needed an additional instrument. The additional instrument consists of two problems used to collect data, namely:

Problem 1. Prove that if $f(x) = x^2$ then $\frac{df(x)}{dx} = 2x$

Problem 2. If $(x) = x^2$, calculate $\frac{df(x)}{dx^3}$

The additional instrument was validated by 2 (two) lecturers of Basic Mathematics I. The instrument was tested on other students (31 students) as a trial test to obtain the instrument's reliability. Each question gave a score of 0-50 points. The reliability test used correlation coefficient test Alpha Cronbach with significance level 95 % and obtained a correlation coefficient $r = 0.59149$ greater than $r = 0.3009$ (from table Alpha Cronbach). So the instrument was reliable (Denzin & Lincoln, 2008)

Analyzing the Data

In this research, the researcher used document data analysis for collecting the data. Document analysis in qualitative research collects and analyzes the data concerning the research by gathering documents, records of interviews, and various materials (Lincoln & Guba, 1985). The data collection is needed for interpretation because the qualitative approach's interpretation is important to a drawn conclusion.

After collecting the quantitative data from the written task, a further stage is to collect the qualitative data through the unstructured interview with the selected subjects. The qualitative data were analyzed by using data collection, data reduction, display data and data verification. All the data collected from the written test were compared with the results of data interviews by meta-analysis. The stages of the meta-analysis are grouping, defining, and determining the connection and consistency among each data, the final conclusion was drawn based on the strong connection among groups of data (Denzin & Lincoln, 2008).

In this study, data analysis was carried out by collecting the results of written answers to problems in the additional instrument, interviewing each subject selected from each type of student's answer, conducting a process of reducing irrelevant data, coding data, mapping data relationships. Data reduction was carried out on data that was not relevant to the focus of the study. Data verification is done by matching written data from the subject's answers, subject interviews, related documents and expert opinions. The process of interpreting data is carried out by understanding the data, linking the data, linking it with related theories, and asking for expert opinion, then drawing conclusions. The meta-analysis process is carried out by comparing the data from existing research results and related concept images to other mathematical concepts.

Data were analyzed using the theory of concept image and concept definition. In this study, concept image is defined as a collection of all mental pictures, processes, and properties associated with the concept of the derivative that is in the students' minds. The components of the concept image are defined, such as in Table 1.

Table 1. Component of Concept Image

Component of Concept Image	Explanation
Mental pictures	All pictures in the minds of the students are used the student to explain the concept or how the students to describe or create the pictures in solving the problem associated with the concept.
Processes	All procedures, ways, or steps are used by the student to describe the concept or in solving the problem associated with the concept.
Properties	All axioms, definitions, lemmas, theorems, or formulas are used by the student to describe the concept or in solving the problem associated with the concept.

Validity and Reliability Analysis

To obtain the validity and reliability of data, researchers used a time triangulation method for all subjects. Three days after the subjects sent their answers via WhatsApp, the researchers asked subjects again to answer similar problems via WhatsApp, such that until consistent data is obtained. Triangulation is a process to obtain the accuracy and consistency of the data. Time triangulation is done by comparing information or data at different times. The basis of this time triangulation is that the data's consistency can be seen at different times in a certain period. If the researcher is still unsure about the consistency of students' answers, the researcher will use triangulation at the different times or another type of triangulation (method triangulation, data source triangulation, or theory triangulation) so that consistent data are obtained.

Findings/Results

Students' Perception of Problem 1

Problem 1 aims to look at students' perceptions in understanding the problem of the derivative of a function. From the perception and their answers to problem 1, it will be investigated their concept image of the derivative. Sixty students have given the answers for problem 1, there exist three categories of student's perceptions to answer problem 1 as follows:

1. Forty-six (76.7 %) students have a perception that the problem should be answered using formula if $f(x) = x^n$ then $\frac{df(x)}{dx} = nx^{n-1}$
2. Ten (17.3 %) students have a perception that the problem should be answered using definition of the derivative as follows: $\frac{df(x)}{dx} = \lim_{y \to x} \frac{f(y)-f(x)}{y-x}$
3. Four (6.7%) students have a perception that the problem should be answered using definition of $\frac{df(x)}{dx} = L$. This means: $\forall \varepsilon > 0, \exists \delta > 0$ such that if $0 < |y - x| < \delta \Rightarrow \left|\frac{f(y)-f(x)}{y-x} - L\right| < \varepsilon$

Students' Conception of Problem 2

Problem 2 aims to evoke mathematical reasoning based on students' concept image of the derivative. In the following step, some ways of how the students solve the problem 2 are categorized. From the participants' written answers were obtained 8 (eight) types of participants' answers {A, B, C, D, E, F, G, H}. Therefore, the researchers selected one participant from each type of participants' answers. So, there are 8 (eight) participants as subjects in this research. To further clarify all subjects' written answers, the researchers conducted written interviews with subjects via WhatsApp. Researchers have conducted in-depth unstructured written interviews via WhatsApp with all subjects related to derived concepts and with the subject's written answers to problems that have been previously given to researchers. So, the statement of the interview results below between the researcher and the subjects is the conclusion made by the researcher from the results of written interviews via WhatsApp with the subjects. So, the researchers obtained 8 (eight) types of the participants' answers as follows:

A. Twenty-one (35 %) of the students change the variable x into x^3 such that the variable in the derivative of a function should be equal, as shown in Figure 3.

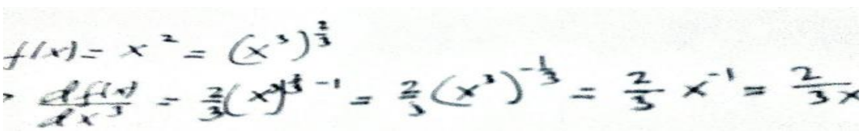
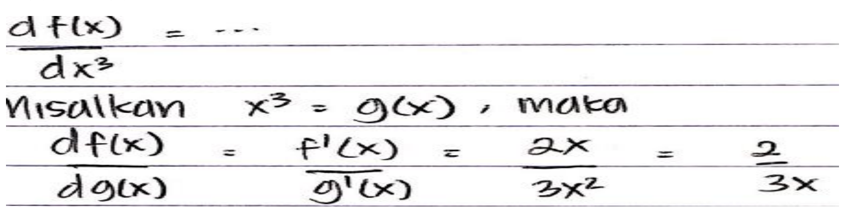


Figure 3. A Student's answer that changes the variable x into x^3

From the written interview, the subject stated that the answer of the problem 2 was a form of substitution of the basic formula of the derivative.

B. Eleven (18.5 %) of the students used a concept of the derivative that is the ratio of the derivative each the function, namely $\frac{df(x)}{dg(x)} = \frac{f'(x)}{g'(x)}$ as shown in Figure 4.



[Let $x^3 = g(x)$, then ...]

Figure 4. A Student's answer used the ratio of the derivative each the function

From the written interview, the subject said that problem 2 can be solved using a quotient of the derivative of two functions.

C. Nine (15%) of the students used a concept of the derivative, that is a limit of the ratio of difference value of the function as shown in Figure 5.

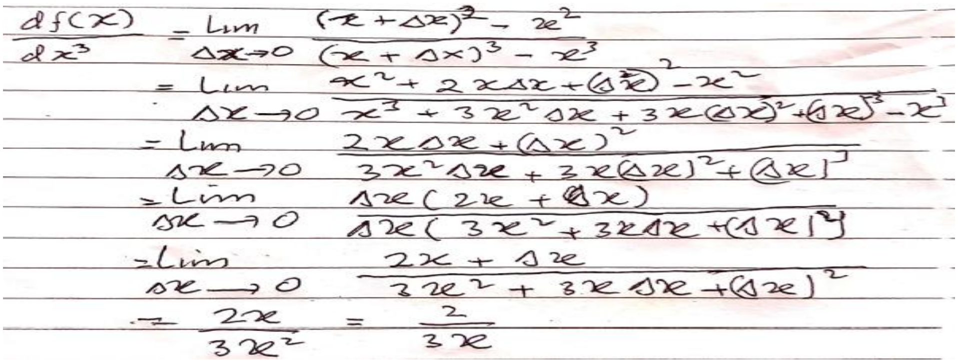


Figure 5. A Student's answer used the ratio of difference value of the function

From the written interview, the subject believed that problem 2 can easily be solved using a definition of the derivative.

D. Four (6.7 %) of the students used a concept of the derivative as a limit of the ratio of change value of the function with change value of *x,* as shown in Figure 6, there is a misconception to the use of the concept of the derivative of a function.

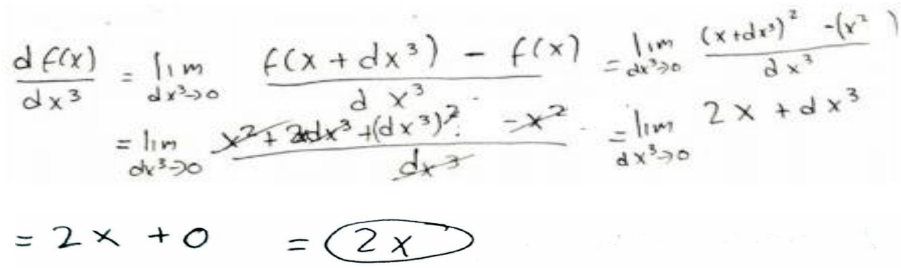
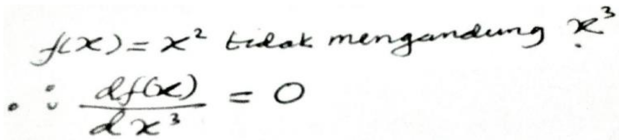


Figure 6. A Student's answer used the concept $\frac{df(x)}{dx} = lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$

From the written interview, the subject claim to be misconception of the definition of the derivative.

E. Five (8.3 %) of the students answered $\frac{df(x)}{dx^3} = 0$, derivative of function $f(x) = x^2$ does not contain variable x^3 as shown in Figure 7, there is a misconception to the use of the properties of the derivative.



[$f(x) = x^2$ does not contain x^3]

Figure 7. A Student's answer that the function $f(x)$ does not contain x^3

From the written interview, the subject believed that because the function does not contain x^3, then the derivative of a function with respect to x^3 is 0. The subject also said that x^2 was assumed as a constant.

F. Four (6.7 %) of the students gave the answers that $\frac{df(x)}{dx^3}$ can't be differentiated of x^3.

[$\frac{df(x)}{dx^3}$ can't be differentiated]

Figure 8.A Student's answer that the function $f(x)$ is not differentiable into x^3

From the written interview, the subject said that function $f(x)$ is only differentiable into x, so derivative of $\frac{df(x)}{dx^3}$ is not exist.

G. Three (8.3 %) of the students used the third derivative of $f(x)$as shown in Figure 9, there is a misperception to the problem 2.

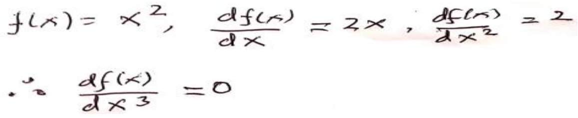


Figure 9. A Student's answer used third derivative of $f(x)$

From the written interview, the subject believed that $\frac{df(x)}{dx^3}$ is as the third derivative of $f(x)$.

H. Three (8.3 %) of the students did not answer problem 2, but they gave only the answer of problem 1 using a basic formula of the derivative.

From the students' answers of problem 1 and problem 2, the distribution of linkage between students' perception of the problem1 with the students' conception of the problem 2 as shown in Table 2.

Table 2. Distribution of Perceptions of the Problem1 with Conceptions of the Problem 2

Students' Conception of Problem 2	**Students' Perception Of Problem 1**			**Total**
	1	**2**	**3**	
A	19	1	1	21
B	11	0	0	11
C	0	6	3	9
D	1	3	0	4
E	5	0	0	5
F	4	0	0	4
G	3	0	0	3
H	3	0	0	3
Total	46	10	4	60
Percentage	76.67	17.3	6.7	100

From Table 2 shows that the conception of 46 students (76.67%) to solve the problem 2 influenced by perception 1 of the problem 1, i.e. they used reasoning that the problem 2 can be solved by using the basic formula $\frac{df(x)}{dx} = nx^{n-1}$ for $f(x) = x^n$. Wherefrom perception 1, the majority of the students (21 students or 35%) used the reasoning of conception A.

From ten students (17.3 %) who have perception 2 of the problem 1, there are 6 students used the reasoning of conception C to solve the problem 2, and from 4 students (6.7 %) who have perception 3 of the problem 1, there are also 3 students used the reasoning of conception C to solve the problem 2.

Category of concept image

From the data analysis, then the researchers decided there exist three categories of students' concept image. These categories are:

1. Symbolically related to a basic formula of the derivative of a function, namely if $f(x) = x^n$ then $\frac{df(x)}{dx} = nx^{n-1}$
2. Symbolically related to limit of the ratio of difference value of a function, namely
$\frac{df(x)}{dx} = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$.
3. Symbolically related to the properties of the derivative.

The students' perception for category (1) was based on their concept image of the derivative, namely that a derivative of a function related to a formula of derivative, namely if $f(x) = x^n$ then $\frac{df(x)}{dx} = nx^{n-1}$. Likewise, for students' perception for category (2), they have a concept image that the derivative of the function is a gradient of tangent line at point x on curve $f(x)$ formulated by $\frac{df(x)}{dx} = \lim_{y\to x} \frac{f(y)-f(x)}{y-x}$. Meanwhile, four students are category (3), they have a concept image that the derivative is a limit of a ratio of the difference, namely $\lim_{y\to x} \frac{f(y)-f(x)}{y-x} = L$. Their answers to problem 1 were based on the experience of how to prove $\lim_{x\to a} f(x) = L$. Results show that different concept images will evoke different mathematical processes. The students of category (a) and (b) only have a perception to answer problem 1, namely how to solve $\frac{df(x)}{dx}$, if $f(x) = x^2$, not to prove why $\frac{dx^2}{dx} = 2x$. This is different for students with category (c), they have a perception that the statement of the symbol in problem 1 is as a concept of the derivative of a function.

Impact concept image on Reasoning

Students' reasoning to solve problem 2 is described as follows: from row 1 (A), row 2 (B), and column 1(1) in Table 2, it shows that perception 1 greatly affects the conception of students to solve the problem 2. It indicates that the mastery of the concept of a basic formula of the derivative by the students strongly affects the student to solve the problem based on the formula. From row 3 (C) and column 2 (2), column 3(3) there exists 9 students who have a concept image of the derivative as a limit of the ratio of difference of function $f(x)$ with respect to x. So the students of category A, B, and C have a true solution. It indicates that teaching the concept of mathematics is less desirable to the students. By using the reasoning Vinner's diagram and Toulmin's diagram, the students' reasoning to solve problem 2 described in Figure 10 as follows:

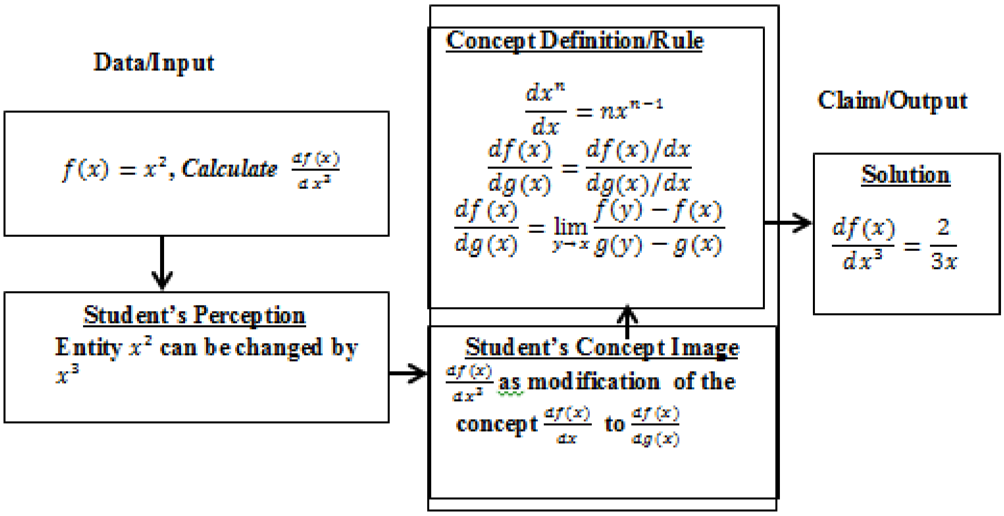


Figure 10. Students' reasoning in solving of problem the derivative of a function for true solution

From row 4 (D) there are 4 students using the concept of the derivative as a limit of the ratio of difference of function and its variable (in perception 2 and 3), but they do not understand the concept. They do not realize that in such a way the solutions will be equal as using $\frac{df(x)}{dx}$ for all functions. However, the results show that students' concept image has a role to build up the conception and reasoning of students. Meanwhile, row 5 (E), row 6 (F), and row 7 (G) show that the students have a concept image of the derivative of a function as a derivative basic formula (column 1). However, they do not understand the notion of the derivative. It indicates that students who know a formula but do not understand the concept, they tend to have a weak concept image of the concept and it seems difficult to establish their mathematical reasoning. Students of category D and G have misperceptions and students of category E and F have misconceptions, this is illustrated in Figure 11. So, students of category D, E, F, and G have the wrong solution. In general, misconception often arises during the initial stages of students learning or in problem-solving. For students in category H, they have perception 1 of problem 1, but they gave irrelevant answers, they are considered not to know how to solve problem 2.

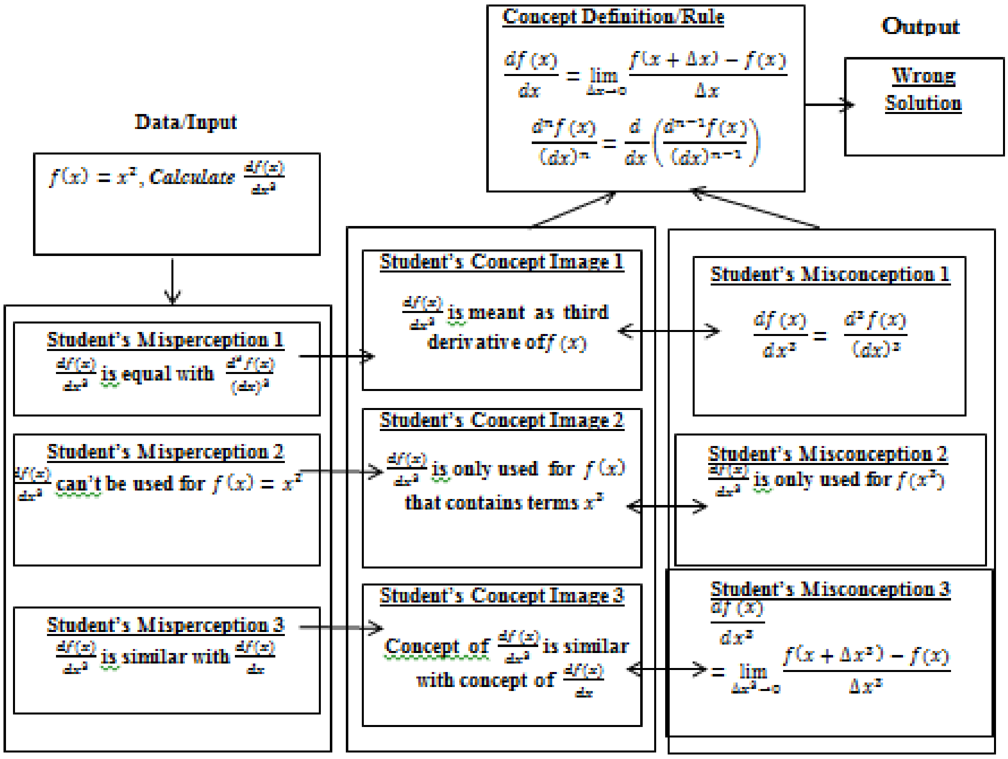


Figure 11. Students' reasoning in solving of problem the derivative of a function for wrong solution

Discussion

The data analysis obtained three categories of students' concept image, namely symbolically related to a basic formula of the derivative of a function, symbolically related to a limit of the ratio of difference value of a function, symbolically related to the properties of the derivative. This is following Tall and Vinner(1981) that the concept image can be in the form of a collection of mental pictures, processes, or properties associated with the concept.

In interviews with the subjects who answered the problem correctly, they generally said that the solution of the problem was based on their previous experiences and also had understood the concept of the derivative. This fact is in line with the results of Vinner and Dreyfus(1989) that a specific student's concept image of a particular concept is influenced by all of this student's experiences associated with the concept. The results show that students' experiences of the derivative become the basis for the formation of their concept image and it becomes its main ability in problem-solving of the derivative of the functions. This inline with Vinner and Hershkowitz (1980), who stated that reasoning in mathematics is not only based on mastery of mathematical definitions and concepts, the more important thing is to have an adequate concept image. Even, Vinner and Dreyfus (1989) stated that students' mathematical reasoning is frequently based on their concept images rather than on a mathematical concept definition.

Some cases appear in the results, some students can't answer the problem correctly due to misperception in the mathematical symbols in the question and misconception to solve the problem. So they have no confidence to solve the problem. Students who have a correct perception and conception of mathematics problems, then their confidence in problem-solving significantly increases (Wismath et al., 2014). When the students were faced with a mathematical problem, then what should be understood statements of the problem as a concept and how the processes used to solve the problem Chin(2008). Chin stated that the statements of a theorem or a mathematical question can be understood as a symbol. This symbol may evoke a proof deduction as a *process*. The general notion of the theorem or question is as a *concept*. Gray and Tall (1994) suggested the notion of the *procept*, which was taken to be characteristic of mathematical symbols. According to Gray and Tall, in a mathematical symbol contains two entities, symbol as process and symbol as a concept.

Conclusions

The purpose of this study is to contribute to the role of concept image in the minds of students on evoking the students in mathematical reasoning. Reasoning in mathematics is an important thing for exploring mathematics. The students are encouraged to always improve their concept image, so that can think and reason in mathematics exceeding their previous owned knowledge. From the results, the students when asked to prove a concept of the derivative of a function (problem 1), 93,3 % of the students have a perception that problem 1 is solved by procedural ability with the simple answer using a basic formula (conception 1 and 2), and only 6.7 % of the students expressed their answer using a conceptual ability. However, upon the discussion above, the students' concept image of the derivative affects the students' mathematical reasoning in problem-solving of the derivative. Students' concept image on the derivative has the role to build up of conception and reasoning of the students in problem-solving of the derivative. The discussions show that if the students have the correct perceptions and conceptions of the problem and have an adequate concept image, then the students will have a great opportunity to build reasoning and solve the problem correctly. Conversely, if the students have a misperception or misconception of the problem or unsuitable with their concept image, then the students will have a wrong solution. In the results, the majority 93.7% of the students (56 out of 60 students) have an ability of the process than the conceptual understanding.

Recommendations

Based on the conclusions of this study, the researchers suggest to the mathematics teachers that the teachers have to give particular attention to the perception of their students to the given problems, and always observe the students' concept image related to activities on the problem- solving. The teachers are expected to improve students' perceptions and the concept images related to the solution of the given problems, such that the students will solve the problem correctly. It is because the level of students' understanding of a mathematical concept can be seen from the level of their concept image of the formal mathematical concept. For further research, suggested how the concept image can influence the students' belief in problem-solving activities of the derivative.

Limitations

This study revealed the concept image only based on participants' written answers to questions in the additional instrument of research, the researchers did not keep an eye on students in answering the questions. The answers were sent via WhatsApp to the researchers. It has not seen yet the overall mental pictures of the participants in their minds related to the concept of the derivative. However, from their concept image based on the participants' written answers, can be obtained some mental pictures, properties, and processes associated with the concept of the derivative. Therefore, the participants' reasoning to solve the problems in the task was limited only to the concept image.

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