Mathematics Education and Graph Theory

PROCEEDINGS OF INTERNATIONAL SEMINAR
ON MATHEMATICS EDUCATION AND GRAPH THEORY

Unit of Publication
Department of Mathematics Education
Faculty of Teacher Training and Education
Islamic University of Malang (UNISMA)
2014

MATHEMATICS EDUCATION AND GRAPH THEORY
Proceedings of International Seminar on Mathematics Education and Graph Theory

These proceedings contain the full texts of paper and talks presented in the International Seminar on Mathematics Education and Graph Theory on June 9, 2014

Reviewers
Surahmat Supangken (UNISMA, Indonesia)
Abdur Rahman As'ari (UM, Indonesia)
Kiki Ariyanti Sugeng (UI, Indonesia)
Sunismi (UNISMA, Indonesia)
Akhsanul In'am (UMM, Indonesia)

Editors
Mustangin
Abdul Halim Fathani

Layouter
Teguh Permadi

First published, 2014
ISBN 978-602-71141-0-4

Published by

Unit of Publication
Department of Mathematics Education
Faculty of Teacher Training and Education
Islamic University of Malang (UNISMA)
Malang, East Java, Indonesia
Phone +62341- 551932, 551822.
Fax +62341-552249
http://www.unisma.ac.id

PREFACE

These proceedings contain the full text of papers and talks presented in the International Seminar on Mathematics Education and Graph Theory. This seminar was held in conjunction with the International Workshop on Graph Masters. The workshop was held on June 7–8, 2014, while the seminar was on June 9, 2014. These events were organized by Islamic University of Malang (Unisma) in cooperation with Indonesian Combinatorial Society (InaCombS).

The workshop and the seminar would not have been possible without the time and energy put forth by the invited speakers. The invited speakers of the workshop were: **Mirka Miller**, University of Newcastle, Australia; **Joseph Miret**, Universitat de Lleida, Spain; **Christian Mauduit**, Institut de Mathematiques de Luminy, France; **Edy T. Baskoro**, Bandung Institute of Technology, Indonesia; **Surahmat Supangken**, Islamic University of Malang, Indonesia; **Tri Atmojo**, State University of Semarang, Indonesia; and **Purwanto**, State University of Malang, Indonesia.

The invited speakers of the seminar were: **Juddy Anne Osborn,** University of Newcastle, Australia and **Abdur Rahman As'ari**, State University of Malang, Indonesia. The seminar was held on the area of mathematics education and graph theory. The main themes of the mathematics education seminar include topics within the following areas (but not limited to): philosophy of mathematics education, curriculum development, learning methods and strategies, learning media, development of teaching material, and assessment and evaluation of learning. The main themes covered in graph theory seminar include topics within the following areas (but not limited to): degree (diameter) problems, ramsey numbers, cycles in graphs, graph labeling, dimensions of graphs, graph coloring, algorithmic graph theory, and applications of graph theory in various fields.

We would like to thank you to the invited speakers and all presenters who have submitted papers, for their valuable and inspiring presentation. A special appreciation goes to: **Surahmat Supangken**, Rector of Unisma and **Kiki Ariyanti Sugeng,** the President of InaCombS, who have made a lot of efforts to prepare this seminar.

We also do not forget to express our gratitude to Islamic University of Malang (Unisma) for providing financial support, and to the Indonesian Combinatorial Society (InaCombS) for the support. We hope that you had a great time and valuable experience during the seminar in Malang.

Malang, July 22, 2014

Editors

TABLE OF CONTENTS

MATHEMATICS EDUCATION:
THEORETICAL (CONCEPTUAL) ARTICLES

 ISBN 978-602-71141-0-4

THE EFFECTS OF REALISTIC MATHEMATICS EDUCATION AND STUDENTS' COGNITIVE DEVELOPMENT LEVELS ON THE UNDERSTANDING OF CONCEPTS AND THE ABILITY IN SOLVING MATHEMATIC PROBLEMS BY JUNIOR HIGH SCHOOL STUDENTS

Sunismi
Department of Mathematics Education, Islamic University of Malang
sunismiunisma@yahoo.com

Abstract

The objectives of this study are: (1) to compare the effectiveness of using *Realistic Mathematics Education* (RME) and conventional approach viewed from the ability in solving mathematic problems by Junior High Schools students grade 8, (2) to examine the effects of different cognitive development phases on the effectiveness of using mathematic learning approach for Junior High Schools students grade 8, (3) to examine the interaction effects between the application of *Realistic Mathematics Education* and conventional learning with cognitive development levels on the ability in solving mathematic problems by Junior High Schools students grade 8. This study employs quasi experiment using *pretest-posttest nonequivalent control group design* to achieve the objectives. To collect data, researcher uses two kinds of instruments, i.e. cognitive development level and the ability in solving mathematic problems. The data are, then, analyzed descriptively and inferentially using *Multivariate Analysis of Variance* (MANOVA) 2x2 factorial design run in SPSS 17.0 *for Windows*. The findings show that (1) the application of RME learning results in learning outcomes of math problem solving better than that of conventional learning, (2) The group of students in transition cognitive development stage have ability in solving math problem better than those who are in concrete stage of cognitive development, and (3) learning approach and cognitive development level indicate that there is no interactive effects on students' ability in solving math problems.

Keywords: *Problem Solving Ability, Realistic Mathematics Education (RME), Cognitive Development Levels*

INTRODUCTION

Learning is a set of activities designed to enable students to learn. Degeng (1998) defined learning as the effort to allow students learn. This effort will let students learn something effectively and efficiently. This definition implies that in learning there are activities of choosing, determining, and developing strategies, methods or learning approaches that will be used to achieve intended learning outcome. These activities are believed to be the primary activity of learning. Thus the natures of learning are planning or designing to allow students to learn. The attention to what students learn is in the scope of curriculum specifying on what goals to achieve and materials to learn to enable students to achieve those goals. Reigeluth (1983) states that learning outcome is related to interaction between method and learning condition including characteristics of learners.

The 2006 regulation of ministry of education no 22 (Depdiknas, 2006) formulates that the instruction of math at Junior High School level should enable students to (1) understand mathematic concepts, (2) use logical reasoning on patterns and characteristics, do mathematic manipulation, (3) solve problems; (4) communicate mathematic ideas; and (5) have positive attitude toward the use of math in daily life. The formulation suggests that students have to be able to solve math problems. Lester (Branca, 1980) confirms that "problem solving is the heart of mathematics", meanwhile Bell (1978) proposes that the ability in solving math problems is seriously needed by society. Because of the importance of math problem solving, problem solving has been a focus in learning math in several counties. The students' ability in problem solving has become a central issue in learning math in America since 1980s (Ruseffendi, 2006) and this also applies to math learning at elementary and Junior high school in Singapore (Kaur, 2004). *National Council of Teachers of Mathematics* (NTCM) (Romberg, 1994) asserts that the ability in solving a problem is one of the important aspects in math learning. This also applies to math learning in Indonesian areas.

This is also supported by Lenchner (1983) stating that solving math problems is a process of applying acquired math knowledge to an unknown new situation. A question will be a problem if the question presents a challenge which can not be solved through a routine procedure known to students. As Cooney, et al. (1975) states "…for question to be a problem, it must present a challenge that can not be resolved by some routine procedures known to the student. On the other hand, Shadiq (2004) classifies problems into two types, i.e. textbook word problems and process problems.

The characteristics of math learning are abstract object, in agreement, deductive reasoning, axiomatic, and structured/gradable so that most students find math difficult and torturing. According Soedjadi (2001a), teachers encounter difficulties in teaching math because of its abstract nature. This presents a challenge for math teachers to reduce its abstractness to make students understand math easier. One of strategies is using concrete objects in teaching math for easier understanding.

The learning of math which has been carried out in schools is mostly unrealistic and not contextual so that students only understand algorithmically by operating formulas they have learnt without understanding the essence of the underlying concepts for each topic. This is contradictory to the 2006 regulation of ministry of education no 22 (Depdiknas, 2006) that "on every occasion, math learning should begin with introducing to contextual problems." This account clearly shows that contextual problem is the heart of math learning. The importance of context is based on a learning paradigm centered on students. One of learning approaches centered on learners is *Realistic Mathematics Education* (RME). This approach makes use of contextual problems as a starting point in learning (de Lange, 1995); Freudenthal, 1991).

Through RME approach, students are expected to understand math concepts easier and develop math concepts they are learning, for the learning process of RME begins with addressing contextual problems relevant to their knowledge and real world to understand math concepts (de Lange 1996; Zulkardi, 2001). Schools really need to apply this approach in math learning as there are many complaints from the students' parents as well as mathematicians on students' low competence in applying math concepts into their daily life.

The results of Suryanto and Somerset's studies (cited in Zulkardi, 2001) in 6 Junior High Schools in several provinces of Indonesia indicate that the results of math test is very low particularly on textbook word problem. Similarly, the reports of *Third International Mathematics and Science Study* (TIMSS) in 2007 (Highlights from TIMSS, 2007) ranks Indonesia 38 out of 48 countries. In addition, PISA (*Program for International Student Assessment*) in 2007 (OECD PISA, 2010) reports a decreasing tendency of Indonesian rank in math competence, 61 out of 65 countries. This further suggests that the Indonesian students' math mastery in international level is considered very low which consequently impact on students' performance. This is caused by several factors including curriculum demanding more on students to achieve the intended target. This means that all materials have to be taught, disregarding students' understanding on math concepts (Marpaung, 2001)

Marpaung (2001), Zulkardi (2001) and Darhim (2004) mention that the students' low learning outcome is because the classroom teachers just give information or still employs conventional approach such as lecture in learning process and the students listen and copy and occasionally

ISBN 978-602-71141-0-4

have question and answer activity followed by examples and practices which less emphasize on logical reasoning and evaluation follows.

This conventional approach is exemplified by two teachers of State Junior High School I ,Dau, and State Junior High School I, Pakis, in Malang regency. In their teaching, they begin their math class with lecture on math materials followed with examples, then, students performed the test exercises as the teachers did. The teachers use that method because if they employ other methods, they need longer time so that they could not finish the materials as prescribed by curriculum. This learning process merely trains students to memorize concepts or procedures and make them unable to solve complex problems. They are like robots that must follow the procedures and thus such learning is considered mechanistic and conventional (Sembiring, Hadi, & Dolk, 2008). Furthermore Romberg (1998) and Armanto (2001) point out that such conventional learning will trap students into memorization, for the math class begins with presenting formulas or concepts, giving sample tests and ending in practices on tests. This approach does not encourage students to construct math concepts, resulting in students' poor logical reasoning. In addition, this approach traps them in abstract concepts so that they can not use math in their real world.

To deal with that problem, the paradigm should be changed from teaching to learning paradigm, which puts emphasis more on active learning process allowing students to discover concepts and do more reflection, abstraction, formalization and application. To make math learning more enjoyable, it ought to be connected to students' experience and real life. Allsopp, Kyger, and Lovin (2007) state that experience or environment–based learning is believed to motivate students, enhance students' understanding and logical reasoning. One of approaches in mathematics learning which employs realistic math is *Realistic Mathematics Education* (Saragih, 2000; Romberg, 1998). This approach is also relevant to the learning paradigm, student-centered rather than teacher- centered.

The RME approach has 3 principles: (1) guided reinvention and progressive mathematizing, giving students opportunities to reinvent concepts or algorithm as mathematically invented; (2) didactical phenomology, providing students with opportunities to invent the concepts themselves by using phenomena implying mathematics concept and real phenomena of their daily life; (3) self developed models stating that the models developed by students should be able to link informal knowledge to formal mathematic knowledge.

The findings of Hadi's study (2002) demonstrates that the application of the RME approach has more positive effects on students of Junior High School, Yojakarta, than that of traditional approach. The positive effects include the increasing students' motivation, activity as well as creativity which are partly because of the inclusion of attractive pictures and stories. The students' ability also progresses especially in understanding mathematics concepts indicated by improving scores from pretest to posttest.

Beside learning approach, another dominant factor affecting students' learning outcome is their logical reasoning which is an inseparable part in learning process particularly in learning math, for mathematics is science acquired through logical reasoning which stresses on rational activities (Sunardi, 2002). Mathematics is constructed as a result of human thought related to ideas, process and logical reasoning. Logical reasoning has specific characteristics closely related to mathematics characteristics, i.e. logical and analytic (Suriasumantri: 2000). Thinking logically means thinking based on certain logics and thinking analytically means that logical reasoning is thinking based on an analysis.

The Piaget's cognitive development theory (1975) has significant influence on education. The stages of Piaget's thoughts have strongly influenced educators in designing curriculum, determining teaching methods and materials. Cognitive development level is the stages of students' cognitive development beginning from childhood to adult, from concrete thinking process to abstract and logic concepts. Jean Piaget is the expert who has conducted numerous studies on development of human cognitive capability. He proposes that human cognitive capability comprises of 4 stages from infant to adult. These stages apply to all ages but in certain period of time, people have different cognitive developments.

The implications of Piaget' theory on cognitive development level on students' guidance to learning mathematics are as follows: (1)mathematics learning is not only outcome-oriented but also mental and thinking process. In this sense, the teachers need to understand the processes on how students come up with correct answers, (2) Students' guidance to learning mathematics can be performed by providing students with opportunities to initiate and involve themselves in learning, (3) the individual differences in terms of progresses are needed in math learning. This theory assumes that all students develop in the same route but with in different paces.

Based on Piaget's cognitive development level, junior high school students who are in teens are in formal operation stage (Slavin, 1994). They certainly have development aspects as teens should be. In this period, the teens also may stay at transition period in thinking process namely from concrete operation stage to formal operation stage in their logical reasoning, meaning that they have not reached the stage of formal thinking yet. The teens begin to realize their thinking limitation, in which they deal with concepts beyond their own experience.

LITERATURE REVIEW

Realistic Mathematics Education (RME)

In mathematic learning, one of the models reflecting constructivism view is *Realistic Mathematics Education* (NTCM, 2000a; de Lange, 1995; & Panhuizen, 1988). The characteristics of learning oriented to RME are: (1) "*reinvention*", requiring students to build a concept and mathematic structure deriving from their own intuition, (2) introduction to concepts and abstraction through concrete objects around them, (3) during matematization, students construct their own ideas which are not necessarily similar to others or to their teacher's, (4) their ideas or thoughts are then confronted to others (Treffers, 1991; Panhuizen, 1998). These characteristics are relevant to constructivism view on learning in which students have to construct their own knowledge resulted from their interaction with their environment as a springboard for new knowledge.

Realistic Mathematics Education (RME) was first developed in Nederland in 1973 (Freudenthal, 1991; de Lange 1995; Panhuizen 1998; Barnes, 2005; Yuwono, 2006). In 1973, Freudenthal introduced an new approach to mathematics learning called *Realistic Mathematics Education* (Freudenthal, 1973; Treffers, 1991). RME has been much determined by freudenthal's view on mathematics. His two important views on mathematics are that mathematics must be connected to reality and mathematics as human activity. In introducing RME, Freudenthal (1973, 1991) assumes that mathematic learning should be connected to reality and mathematics as human activity. This implies (1) mathematics must be close to students and connected to their daily life, (2) mathematics as human activity meaning that the students should be given opportunities to perform mathematization activities for each mathematic topic. This could be done through exploration of various situations and contextual problems. Reality, in this sense, also refers to any situation which can be imagined by students (Gravemeijer, 1994).

Realistic Mathematics Education (RME) is an approach to learning mathematics using contextual and real life problems in acquiring and developing mathematic concepts by students (de Lange, 1995 & Panhuizen, 1998). Further, de Lange (1995), Traffers (1991) state that RME is a learning referring to social constructivism specialized for mathematic

ISBN 978-602-71141-0-4

education. In similar tone, Soedjadi (2001) points out that RME approach makes use of reality and students' surrounding objects to facilitate learning process to achieve mathematic objectives better than the past.

Stages of Cognitive Development

Piaget's theory on cognitive development is one of the theories which accounts for how children adapts to, interpret objects and events around them such as how children learn characteristics or functions of objects like toys, furniture, food and other social objects including themselves, parents and friends. This theory also explains how children categorize objects to find out their similarities and differences, understand the causes of changes in objects and events as well as to predict those events.

Piaget mentions that cognitive structures are as schemas, sets of schema. An individual is able to remember, understand and respond to stimuli because of these schemas. If the schemas possessed by children are able to account for what they perceive from the environment, this is called equilibrium but if they encounter new situations which can not be explained by existing patterns, they will experience disequilibrium sensation, an unfavorable condition. This schema development progresses as a result of their adaptation to environment. Adaptation (functional structure) is a term employed by Piaget to show the importance of the patterns of individual's relationship with his environment in cognitive development processes. Still, according to Piaget, this adaptation consists of two complementary processes, *assimilation* and *accommodation*. Assimilation is an integration between external elements and complete structure of organism. Meanwhile accommodation is creating new step or combining old terms to face new challenges.

Cognitive development is the change from possessed equilibrium to newly acquired equilibrium. Piaget states that there are 4 factors in relation to intellectual development namely (1) maturity, (2) physical experience, (3) social interaction, and (4) equilibration process (self regulation process). These 4 factors are the prerequisite for student cognitive development (Slavin, 1994).

Moreover, Piaget proposes cognitive development experienced by each individual in more detailed, from infant to adult. This theory is structured based on clinical studies to children with different ages in Swiss. Based on his findings, Piaget proposes that there are 4 stages of cognitive development of each individual progressing chronologically or subsequently , i,e. (a) motor sensory: 0-2 years old, (b) pre operational: 2-7 years old, (c) concrete operational: 7-11 years old, and (d) formal operation: 11-15 years old (Slavin, 1994)

The ability of problem solving

Problem solving is a primary focus in mathematic learning. This is obviously expressed in the formulation of competence standard of mathematic learning as prescribed by lesson unit-based curriculum or KTSP (Depdiknas, 2006; Pusat Perbukuan, 2005) affirming that problem solving is the main focus in mathematic learning either closed-ended problem with a single solution, open-ended problem with multiple solutions, or problems with various ways of solution. This is reconfirmed by the 2006 regulation of ministry of education no 22 on content standard for elementary and junior high school concerning mathematic learning that (1) mathematic learning should be focused on problem solving either closed-ended problem with a single solution, open-ended problem with multiple solutions, or problem with various ways of solution, (2) in every occasion , mathematic learning should begin with introducing contextual problems.

The explanation above demonstrates that the ability to solve problems is a primary objective in mathematic learning. Therefore the mathematics teachers ought to understand purpose of solving mathematic problem and sharpen their skills to help students solve mathematic problems. According to Holmes (1995) the underlying reason why someone needs to learn solving mathematic

ISBN 978-602-71141-0-4

problems is based on the fact that the one who is able to solve problems will live productively in this 21st century. He also states that someone who skillfully solves the problem will be able to survive well, be a productive worker, and understand complex issues related to society. Similarly, Utari (1994) defines problem solving as the ability to solve complex and atypical problems, to apply mathematics to daily life or other situation, to prove or create and test a conjecture.

According to Lechner (1983), solving mathematical problem is a process of applying acquired mathematic knowledge to an unfamiliar, new situation. On the other hand, Robert Harris in his site www.visualsalt.com. (accessed on October 2, 2010) asserts that solving a problem is "the management of a problem in a way that successfully meets the goals established for treating it".

In general, the steps of an approach to problem solving refers to a- four step model of problem solving proposed by George Polya. While Polya's theory (Polya, in Zhu, 2007) defined mathematical problem solving as a process that involved several dynamic activities: understanding the problem, making a plan, carrying out the plan and looking back. The latter definition is applied to the discussion in this review. In his book, "*How to solve it*", Polya proposed 4 steps of problem solving in math i.e. (1) understanding the problem, (2) devising plan, (3) carrying out the plan, and (4) looking back (Alfeld, 1996). These 4 steps of problem solving are believed to be the most popular strategy (Holmes, 1995) which explicitly elaborates the steps very applicable to solve routine or non-routine problems.

METHOD

This study employs quasi experiment using 2x2 factorial version of *pretest-posttest nonequivalent control group design* (Ardhana, 1987; Kerlinger, 2000; Tuckman, 1999). Quasi experiment design is typically used in experimental studies in classroom which use intact group due to technical issue, in which randomization can not be performed. The unequal group in nonequivalent design becomes the drawback in determining cause-and-effect (Salkind, 2006).

Based on the design, the graph is illustrated as follows:

Fig. 1

O_1	X_1	Z_1	O_2
O_3	X_2	Z_1	O_4
O_5	X_1	Z_2	O_6
O_7	X_2	Z_2	O_8

Notes:

O_1, O_3, O_5, O_7	: Observation of pretest result
O_2, O_4, O_6, O_8	: Observation of posttest result
X_1 dan X_2	: Treatment using RME and control with conventional approach
Z_1 dan Z_2	: The levels of cognitive development (concrete and transitional stages)
…………….	: Intact group

Fig. 1: The Diagram of Quasi Experiment of Pretest-Posttest Nonequivalent Control Group Design

The subjects in this study are students of Junior High School 1, Dau, Junior High School 1, Pakis, grade 8 semester 2 in Malang Regency. This study includes 4 classes of subject group. To choose the 4 classes as subjects of this study, the researcher employs *cluster random sampling* technique (Ardhana, 1988, Long, 1986). The number of the subjects is 148

ISBN 978-602-71141-0-4

students comprising of 71 males and 77 females.

Variables in this study are *the approaches to learning mathematics* (RME and conventional approaches) as independent variables, the *levels of cognitive development* (concrete and transitional stages) as moderating variable, and *the ability to solve problem* as dependant variable. The categorization of students' cognitive levels is based on the results of formal logical reasoning test (TKBF) which has been developed by Burney and was translated and adapted by Ardhana (1983) for Indonesian students. This test reliability has been tested using Cronbach's Alfa and Corrected Item-Total Correlation for each item. The result of trial test for TKBF has been proven valid and reliable. Meanwhile the classification to determine the levels of students' cognitive development uses Burney classification model (in Ardhana, 1983) as seen in the following table.

Table 1: Classification of Cognitive Development Levels

Total score	Classification of Cognitive Development Levels
0 - 10	Concrete phase
11 - 16	Transition, from concrete to formal phase
17 - 24	Formal phase

The instrument of dependant variable is essay test administered in pre and post test. The researcher also performs content validity and expert judgment tests to find out the relatedness between essay test and learning objectives. The review of the two experts says that each item of the essay test on the ability to solve problems has a very good relevance to leaning objectives and has good content validity. Thus each item of the essay test on the ability to solve problems has met the criteria of content validity so that it can be properly used to measure students' learning outcome.

In this study, the researcher analyzes the data descriptively and inferentially using *Multivariate Analysis of Variance* (MANOVA), 2x2 factorial version. The descriptive analysis is to describe students' learning outcome (the ability to solve problems) and Standard Deviation (SD). The researcher utilizes SPSS 17.00 for Windows to analyze the data. In addition, the researcher has set a 5% level of significance to test null hypothesis (Ho). Earlier, parametric assumption test to see the normality of data distribution using *Kolmogorof-Smirnov* and Variance homogeneity test among groups using *Levene' Test of Equality of Error Variances* were performed.

RESULT

The result of parametric assumption test shows that the posttest (problem solving) scores of groups in concrete and transition phases for all treatment groups have normal distribution or satisfy normality test.

The variance homogeneity test among groups employing *Levene' Test of Equality of Error Variances* with *Box's M Test* method and variance homogeneity test using *Levene's test* show that variance homogeneity assumption test is satisfied i.e. the variance of individual dependant variable is homogenous among treatment groups, both learning approach treatment and cognitive development levels.

To test hypothesis, the researcher employs *Multivariate Analysis of Variance* (MANOVA) 2 x 2 factorial run in SPSS 17.00 for Windows with 5% level of significance. The result of MANOVA test consists of two parts; (1) the results of multivariate test to find out whether there are differences in independent variable collectively in treatment group, and (2) results of test of between subject effects to see whether there is difference in independent variable individually in treatment groups. The results of multivariate test are shown in the Table 2.

Table 2: The results of Factorial Multivariate test

Multivariate Tests

Effect		Value	F	Hypothes is df	Error df	Sig.	Partial Eta Squared
Intercept	Pillai's Trace	.973	2463.493[a]	2.000	138.000	.000	.973
	Wilks' Lambda	.027	2463.493[a]	2.000	138.000	.000	.973
	Hotelling's Trace	35.703	2463.493[a]	2.000	138.000	.000	.973
	Roy's Largest Root	35.703	2463.493[a]	2.000	138.000	.000	.973
Approach	Pillai's Trace	.243	22.171[a]	2.000	138.000	.000	.243
	Wilks' Lambda	.757	22.171[a]	2.000	138.000	.000	.243
	Hotelling's Trace	.321	22.171[a]	2.000	138.000	.000	.243
	Roy's Largest Root	.321	22.171[a]	2.000	138.000	.000	.243
Cognitive development levels	Pillai's Trace	.257	23.806[a]	2.000	138.000	.000	.257
	Wilks' Lambda	.743	23.806[a]	2.000	138.000	.000	.257
	Hotelling's Trace	.345	23.806[a]	2.000	138.000	.000	.257
	Roy's Largest Root	.345	23.806[a]	2.000	138.000	.000	.257
Approach * Cognitive development levels	Pillai's Trace	.106	8.147[a]	2.000	138.000	.000	.106
	Wilks' Lambda	.894	8.147[a]	2.000	138.000	.000	.106
	Hotelling's Trace	.118	8.147[a]	2.000	138.000	.000	.106
	Roy's Largest Root	.118	8.147[a]	2.000	138.000	.000	.106

a. Exact statistic

b. Computed using alpha = ,05

c. Design: Intercept + Approach + Cognitive Development Levels + Cognitive development levels * Cognitive Development Levels

Table 2 shows that out of 4 types of multivariate test methods, *Pillai's Trace, Wilks' Lamda, Hotelling's Trace and Roy's Largest Root,* have the same significance values for all tested variables. The significance value of learning approach, of cognitive development level, and interaction of learning approach and cognitive development levels show less than 0, 05. This means (a) in posttest, the ability to solve problems is collectively different as obviously seen in groups of learning approach (RME and conventional), (b) in posttest, the ability to solve problems is collectively different as obviously seen in Cognitive Development Levels (TPK) (transition and concrete phases), and (3) there is obvious, collective interaction between the learning approaches and cognitive development levels with the ability to solve problem in posttest.

To find out effect test among variable is important in which it will be used to test null hypothesis. The outcome of effect test among variables is presented in Table 3.

ISBN 978-602-71141-0-4

Table 3. The effects among variables (inter-variable effects)

Tests of Between-Subjects Effects

Source	Dependent Variable	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Corrected Model	The ability in solving problems	14715.714[c]	3	4905.238	13.741	.000	.229
Intercept	The ability in solving problems	451339.993	1	451339.993	1264.335	.000	.901
Approach	The ability in solving problems	5145.379	1	5145.379	14.414	.000	.094
TPK	The ability in solving problems	8564.428	1	8564.428	23.991	.000	.147
Approach * TPK	The ability in solving problems	579.416	1	579.416	1.623	.205	.012
Error	The ability in solving problems	49619.982	139	356.978			
Total	The ability in solving problems	505507.692	143				
Corrected Total	The ability in solving problems	64335.697	142				

a. R Squared = ,357 (Adjusted R Squared = ,343)
b. Computed using alpha = ,05
c. R Squared = ,229 (Adjusted R Squared = ,212)

Referring to Table 3, it can be concluded that (1) there is a significant difference in the Junior High School students' ability of grade 8 in solving mathematic problems between those who learn through RME and convention learning approaches, (2) there is a significant difference in the Junior High School students' ability of grade 8 in solving mathematics problems among students with different cognitive development levels (students in transition and those in concrete stages), and (3) there is no interaction between mathematics learning approaches (RME and conventional) and the cognitive development levels (transition and concrete levels) with the Junior High School students' ability of grade 8 in solving mathematic problems.

The more comprehensive description on interaction patterns between learning approaches and cognitive development levels with the ability in solving problems in posttest can be seen in Fig. 2

ISBN 978-602-71141-0-4

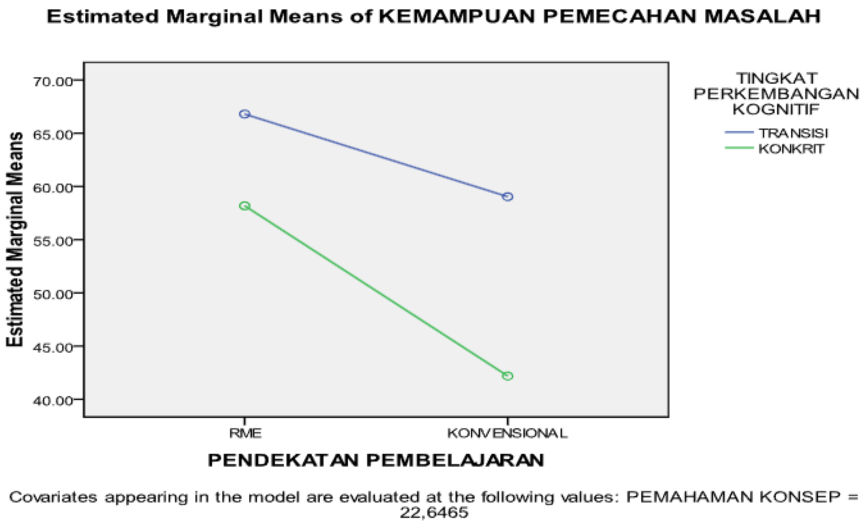

Fig 2. Interaction Patterns between Learning Approaches and Cognitive Development Levels with the Ability in Solving Problems in Posttest

DISCUSSION

Through RME learning approach, mathematics is connected to students' real world. In addition, students need to be involved actively since mathematics is human activity. This can be realized through problem solving activities relevant to students' real life. If the students get used to solving contextual problems, the teachers will get advantages as follows: (1) they can educate their students to be good problem solvers since problem solving is crucial in mathematics education, (2) the learning process should begin with realistic problems facilitating and guiding students to problem solving, (3) the realistic problems will motivate students better to solve problems, and (4) the realistic problems allow students to work at their logic levels and thus facilitating them to comprehend mathematic concepts.

The use of contextual problems in RME approach can be used as inspiration to understand problems, interpret reality, and as thinking activities. This kind of learning is intended to lead students to construct concepts and structure models from their real life, enabling them to deal with real problems. Mathematic learning is human activity; therefore, the learning should center around their daily life. Moreover, students should be given opportunity to learn inventing and performing mathematizing activities in each mathematic topic. This can be done by providing students with contextual or realistic problems or something which can be imagined by students (Gravemeijer, 1994).

In contrast to RME learning, in conventional learning, teachers put more emphasis on the skills to solve test items and memorization. This model of learning, according to Marpaung (2002) will restrict students from getting familiar with alternative solutions and students will much depend on teacher thus they do not get used to trying alternatives to solve problems effectively and efficiently.

In conventional learning, students are also used to obtaining information quickly through listening, reading for information, experimenting based on received information, and answer related questions quickly. Conventional learning as information assimilation has characteristics as follows: (1) obtaining information quickly, (2) organizing of information becomes general principles, (3) the use of general principles to deal with specific cases. The key feature of conventional learning in on information source which is symbolic including listening to teacher's

 ISBN 978-602-71141-0-4

explanation or reading information source greatly influences learning processes.

The students' cognitive development progresses from concrete to formal stages. This in line with Bruner (Hudojo, 2005) students' cognitive development moves from enactive, iconic and symbolic. Because mathematics also deals with abstract concepts and deductive reasoning, it will not be appropriate if the learning is performed in other way around, i.e. from formal to concrete. The abstractness of mathematics needs to be visualized to the more concrete so that the students in formal reasoning stage are able to comprehend mathematics easily. This can be carried out by using meaningful reality or students' environment or surrounding. Meaningful reality can be anything which can be understood by students through direct observation or imagination. On the other hand, meaningful environment is everything around students' life either school environment, family or society.

The ability of students' thinking at concrete and transition levels is truly limited as their thinking is still closely related to concrete and semi-concrete objects; therefore, they are not able to perfectly think deductively and tend to think inductively. As a result, to inculcate the mathematic concepts, the teachers should use concrete objects or relate those concepts to contextual problems known to students for better comprehension. The abstraction follows after the students understand the mathematic concepts through those concrete objects. The abstraction model depends on students' readiness and ability, the higher the level of cognitive development, the more formalized the concepts should be.

The ability in solving problems explicitly has become the objective of mathematic learning and been expressed in curriculum. The underlying reasons of teaching mathematic problems are: (a) problem solving develops cognitive skills in general, (b) problem solving results in creativity, (c) problem solving is part of a mathematic application process, and (d) problem solving motivates students to learn mathematics. It can be inferred that learning solving problems is one of the methods to encourage creativity as a product of students' thinking. The ability in solving problems is a capability as a result of high level cognitive learning as reasoning and intellectual skills. Intellectual skill at problem solving skill, students are demanded to able to identify, understand problems and be good at choosing, using, developing, and organizing high level rules related to their uses followed by analysis and inferring processes.

In this study, the students who have reached transition level of their cognitive development level earn average score higher that those at concrete level in mathematic problem solving. This further suggests that the students who have reached higher cognitive development level, transitional level but have not reached formal level, will be able to solve mathematic problems better than those in concrete level. Although the students at transition level have not reached deductive thinking perfectly, their reasoning has embarked on formal operational stage, enabling them to use their intellectual skill to solve problems. In the meantime, students at concrete reasoning level still encounter difficulties in solving mathematic problems because the students at this level have ability just to use logic operation in solving problems but to solve those related to concrete objects or events but unable to solve hypothetical and wholly verbal problems or problems which need more a complex operation.

Based on the explanation above, RME approach gives the greatest advantages to students whose cognitive development level is at transition level although have not reached formal level perfectly yet, but their reasoning has embarked on formal operation, allowing to be able to perform activities in RME learning processes. The students at the formal stage or at transition stage whose reasoning is close to formal stage reasoning have the following abilities (1) hypothetic-deductive reasoning, (2) thinking using proportion and ratio, (3) controlling variables, (4) using probability, (5) using combinatorial logic, and (6) abstract reasoning. Those abilities are needed to develop ability in solving problems (Flavell in Ardhana, 1983).

The students reaching transition phase, from concrete to formal stages, also have abilities close to the abilities of those in formal stage implying that they are independently able to invent concepts and solve problems which further suggest that the ability of students at this phase is relevant to RME learning approach which demand the students to explore, create and develop their thinking independently to solve problems. Hence, there is significant interaction between RME learning approach and cognitive development at transition stage.

There is no interaction between mathematic learning approach (RME and conventional) and cognitive development level (transition and concrete stages) to students' ability in solving mathematic problems by Junior High School students of grade 8. No interaction indicates that the levels of students' cognitive development do not give effects on relation between learning approaches (RME and conventional) to students' ability in solving mathematic problems. This suggests that either groups of students at transition and concrete levels earn high score on problem solving both those who study using RME approach and conventional approach even though students' scores in concrete level are not as high as those in transition level. These findings corroborate the finding of the 4th study stating that there is a significant difference in problem solving ability of Junior High School students of grade 8 taught using different learning approaches (RME and conventional). This also means that learning approach gives significant effect on the students' ability in solving mathematic problems. This account is also supported by theoretical review as proposed by Freudenthal (1991), Panhuizen (1998), and Soedjadi (2001b) pointing out that RME learning is relevant to mathematics as human activity is viewed as activities to find and solve problems as well as organizing subject matter so that this kind of learning is believed to facilitate mathematic learning process to achieve the objectives of mathematic education well.

CONCLUSIONS AND SUGGESTIONS

Conclusions

Based on general descriptions, hypothesis testing and discussion, the researcher draws conclusions from this study:

1. The use of RME learning gives students ability in solving mathematic problems better than that of conventional learning.
2. The group of students at transition stage of cognitive development (from concrete to formal stages) have the ability in solving mathematic problems better than those who are at concrete stage of cognitive development
3. Learning approaches and level of cognitive development show no interactive effect on the students' ability in solving mathematic problem. No interaction means that the levels of students' cognitive development do not give effect on the relation between learning approaches (RME and conventional) and students' ability in solving mathematic problems.

Suggestions

For dissemination and making use of these research findings, the researcher offers the following suggestions: (1) RME learning is more effective to improve students' ability in solving mathematic problems than conventional learning. This is because the students invent the mathematic concepts by themselves with the help of their teacher. This encourages students to be active, creative and courageous to express their ideas; meanwhile the teacher serves as a facilitator, motivator and guide when students solve problems, (2) Students of grade 8 can achieve ability in solving mathematic problem optimally when mathematic teachers use RME approach by considering the levels of students' cognitive development as alternatives to learning facilitation, (3) in presenting contextual mathematic texts or problems, the teachers employing RME approach need to pay

ISBN 978-602-71141-0-4

attention to differences in the levels of students' cognitive development, whether they are at the level of concrete operation or transition to allow them to understand the problems easily.

SPECIAL THANK

I would like to express my gratitude to Islamic University of Malang for giving me opportunities to conduct this study.

REFERENCES

Allsopp, D. H, Kyger, M. M, dan Lovin, L. D. 2007. *Teaching Mathematics Meaningfully : solution for reaching struggling learners*. Paul. H. Brookes Publishing. London.

Ardhana, W. 1983. *Kesanggupan Berpikir Formal A La Piaget dan Kemajuan Belajar di Sekolah.* Disertasi tidak diterbitkan. Malang: PPs IKIP Malang.

Armanto, D. 2001. *Aspek Perubahan Pendidikan Dasar Matematika melalui Pendidikan Matematika Realistik (PMR).* (Kumpulan makalah pada Seminar Nasional Sehari: Penerapan Pendidikan Matematika Realistik Pada Sekolah Dan Madrasah). Medan: tidak diterbitkan.

Bakker, A. 2004. *Design Research in Statistics Education: On Symbolizing and Computer Tools.* Utrecht: Freudenthal Institute.

Barnes, H. 2005. *The theory of Realistic Mathematics Education as a Theoretical Framework for Teaching Low Attainers in Mathematics.* University of Pretoria: Pythagoras 61, June, 2005, pp. 42-57

Bell, F.H. 1978. *Teaching and Learning Mathematics (in Secondary Schools).* Second Printing. Dubuque, Iowa: Wm. C. Brown. Company.

Branca, N.A. 1980. "Problem Solving as a Goal, Process, and Basic Skill". *Problem Solving in School Mathematics.* Editor: Krulik, S. and Reys, R.E. Reston: NCTM.

Cooney, T.J., Davis, E.J., Henderson, K.B. 1975. *Dinamics of Teaching Secondary School Mathematics.* Boston: Hougthon Mifflin Company.

Darhim. 2004. *Pengaruh Pembelajaran Matematika Kontekstual Terhadap Hasil Belajar dan Sikap Siswa Sekolah Dasar Kelas Awal dalam Matematika.* Disertasi tidak diterbitkan. Bandung: PPS UPI.

Devrim. 2006. Attitudes of 7th Class Students Toward Mathematics in Realistic Mathematics Education. *International Mathematical.* Forum, 1, 2006, no. 39. Turkey: Department of Mathematics Education Balıkesir University.

Degeng, I N. S, 1997. *Strategi Pembelajaran Mengorganisasi Isi dengan Model Elaborasi.* Malang: Penerbit IKIP Malang.

Degeng, I N. S, 1998. *Mencari Paradigma Baru: Pemecahan Masalah Belajar dari Keteraturan Menuju Ke Kesemrawutan.* Makalah Pidato Pengukuhan Guru Besar IKIP Malang.

de Lange, J. 1995. Assessment: No Change Without Problems, In Romberg, T.A. (ed). *Reform in School Mathematucs and Authentic Assessment.* New York: Suny Press.

de Lange, J. 1996. Using and Applying Mathematics in Education. In A.J. Bishop, K. Clements, Ch. Keitel, J. Kilpatrick & C. Laborde (Eds.), *International Handbook of Mathematics Education.* Dordrecht: Kluwer Academic Publishers.

de Lange, J. 2004. *Mathematical Literacy for Living from OECD-PISA Perspective.* Paris: OECD-PISA.

Depdiknas. 2006. *Permendiknas Nomor 22 Tahun 2006 Tentang Standar Isi Sekolah Menengah Atas.* Jakarta: Depdiknas.

Fauzan, A. 2002. *Applying Realistic Mathematics Education in Teaching Geometry in Indonesian Primary Schools.* Doctoral Dissertation, University of Twente, Enschede, The Netherlands.

Freudenthal, H. 1991. *Revisiting Mathematics Education.* China Lectures. Dordrecht: Kluwer Academic Publishers.

Gagne, R. M. 1977. *The Conditions of Learning Third Edition*. New York: Holt, Reinhart, and Winson.

Gravemeijer, K.P.E. 1994. *Developing Realistic Mathematics Education*. Utrecht: CD-ß Press/Freudenthal Institute.

Hadi, S. 2005. *Pendidikan Matematika Realistik dan Implementasinya*. Banjarmasin: Tulip.

Hadi, S. 2002. *Effective Teacher Professional Development for the Implementation of Realistic Mathematics Education in Indonesian*. Doctoral Dissertation, University of Twente, Enschede, The Netherlands.

Highlights From TIMSS. 2007. *Mathematics and Science Achievement of U.S. Fourth and Eight-Grade Students in an International Context*. IES National Center for Education Statistics. (Online), http://nces.ed.gov/pubs2009/2009001.pdf, diakses 20 Maret 2009

Hudojo, H. 2005. *Kapita Selekta Pembelajaran Matematika*. Malang: UM Press.

Kerlinger, F. N. 2000. *Asas-asas Penelitian Behavioral*. Terjemahan: Foundation behavioral research, oleh: Simatupang, L. R., & Koesoemanto, H. J. Yogyakarta: Gadjah Mada University Press.

Lenchner, George. 1983. *Creative Problem Solving in School Mathematics*. New York: Glenwood Publication Inc.

Longworth, N. 1999. *Making Lifelong Learning Work: Learning Cities for a Learning Century*. London: Kogan Page.

Marpaung, Y. 2002. *Implementasi Pendidikan Matematika Realistik di Indonesia.* (Kumpulan makalah pada Seminar Nasional Sehari: Penerapan Pendidikan Matematika Realistik Pada Sekolah Dan Madrasah). Medan.

Mullis, I.V.S., Martin, M.O., Gonzales, E.J., Gregory, K.d., Garden, R.a., O'Connnor, K.M., Chrostowski, S.J., and Smith, T.a. 2000. TIMSS 1999: *International Mathematics Report. Boston: The International Study Center, Boston College, Lynch School of Education.*

NCTM (*National Council of Teachers of Mathematics*). 2000b. *Learning Mathematics for a New Century, 2000 Yearbooks NCTM*: Reston VA.

Panhuizen, Van Den Heuvel and Wijers. 2005. *Mathematics Standards and Curricula in the Netherlands*. ZDM 2005 Vol. 37 (4).

Panhuizen, Van Den Heuvel. 2003. *The Didactical Use of Models in Realistic Mathematics Education*: An Example from a Longitudinal Trajectory on Percentage. Educational Studies in Mathematics, 54(1), 9-35.

Panhuizen, Van den Heuvel. 1998. *Realistic Mathematics Education Work in Progress*. http://www.fi.nl/ ……2000. *Mathematics Education in the Netherlands a Guided Tour*. (Online) http://www.fi.uu.nl/en/indexpulicaties.html, diakses 24 Mei 2010

Piaget, J and Inhelder, B. 1975. *The Origin of the Idea of Change in Children*. L.Leake, P.Burrell, & H. Fishbien (trans). London: Routledge & Kegan Paul.

Polya, G. 1973. *How To Solve It* (2nd Ed). Princeton: Princeton University Press.

Reigeluth, C.M. 1983. *Instructional-Design Theories and Models; an Overview Their Current Status.* London: Lawrence Erlbaum Associates, Publisher.

Romberg, T.A. (1994). *Classroom Instruction that Foster Mathematical Thinking and Problem Solving: Connections between Theory and Practice*. In *Mathematical Thinking and Problem Solving.* Editor: Schoenfeld, A.H. Hove: Lawrence Erlbaum Associates.

Romberg, T.A. 1998. Problematic Features of the School Mathematics Curiculum. In Bishop et al. (Eds.). *International Handbook of Mathematics Education*. Edited Dordrecht, The Netherlands: Kluwer Academic Publishers.

Ruseffendi, E.T. 2006. *Pengantar Kepada Membantu Guru Mengembangkan Kompetensinya*

 ISBN 978-602-71141-0-4

dalam Pengajaran Matematika untuk Meningkatkan CBSA. Bandung: Tarsito.

Saragih, S. 2000. Analisis Strategi Kognitif Siswa SLTP Negeri 35 Medan dalam Menyelesaikan Soal-soal Matematika. *Jurnal Penelitian Kependidikan* No. 2, Des 2000. Malang.

Selter & Becher. 1996. Elementary School Practice. In A.J. Bishop. *International Handbook of Mathematics Education*. Dordreccht: Kluwer.

Sembiring, R. Hadi, S. dan Dolk, M. 2008. *Reforming Mathematics Learning in Indonesian Classrooms Through RME*. *ZDM Mathematics Education* (2008) 40:927–939 DOI 10.1007/s11858-008-0125-9.

Shadiq, F. 2004. *Pemecahan Masalah, Penalaran dan Komunikasi.* Makalah disajikan dalam diklat instruktur/Pengembang Matematika SMA Jenjang Dasar di PPPG Matematika Yogyakarta.

Simatwa, Enose M.W. 2010. Piaget's Theory of Intellectual Development and Its Implication for Instructional Management at Pre-Secondary School Level. *Educational Research and Reviews*. v5 n7 p366-371 Jul 2010.

Slavin, R.E. 1994. *Educational Psychology: Theory and Practice.* 5th edition. Boston: Allyn and Bacon.

Snelbecker, Glenn. E,. 1974. *Learning Theory, Instructional Theory, and Psychoeducational Design,* New York: McGraw-Hill Book Company.

Soedjadi R. 2001a. *Pemanfaatan Realitas dan Lingkungan Alam Pembelajaran Matematika*. (Makalah disampaikan pada Seminar Nasional tentang Realistic Matematic Education Universitas Negeri Surabaya).

Soedjadi R. 2001b. *Kiat Pendidikan Matematika di Indonesia.* Jakarta: Departemen Pendidikan Nasional.

Treffers, A. 1991. Realistic Mathematics Education in the Netherlands 1980-1990. In L. Streefland (Ed.), *Realistic Mathematics Education In Primary School*. Utrecht: CD-B Press/Freudenthal Institute

Tuckman, B.W. 1999. *Conducting Educational Research.* Fifth Edition: Orlando, Harcourt Brace College Publisher.

UU RI Nomor 20 Tahun 2003 *Tentang Sistem Pendidikan Nasional*. Jakarta.

Zulkardi. 2001. *Efektiviitas Ligkungan Belajar Berbasis Kuliah Singkat dan Situs Web sebagai suatu Inovasi dalam Menghasilkan Guru RME di Indonesia.* Makalah disajikan pada seminar nasional "Pendidikan matematika realistik Indonesia" tanggal 14-15 November 2001 di Universitas Sanata Dharma, Yogyakarta.

Zulkardi. 2002. *Developing A Learning Environment on Realistic Mathematics Education for Indonesian Student Teachers.* Doctoral Dissertation, University of Twente, Enschede, The Netherlands.